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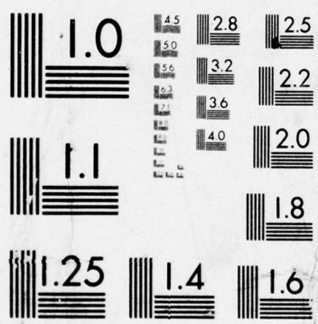

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On Impulse Response Data and the Uniqueness of the Inverse Problem

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Abstract. The problem of reconstructing the density and elastic properties of a slab of infinite horizontal extent and finite thickness set in motion by a double couple excitation is considered. It is shown that the information contained in the amplitude and frequency response associated with a single horizontal wave number is sufficient to insure the uniqueness of the solution of this inverse problem.

Key words: Inverse problem - Amplitude and frequency response.

1. Introduction

The present paper is concerned with the uniqueness of the solution of the inverse problem for the internal structure of the earth.

The aim is to find what data are required in order to find both the Lamé parameters and the density uniquely. These data should be such as to enable us to perform the inversion under the following two conditions:

(i) the solution should be obtained ab initio and should not be a perturbation of an existing earth model;

(ii) an ad hoc assumption à la Adams-Williamson should not be used, but rather its validity should be checked.

I have been able to make some progress in answering the question of the necessary data for problems simpler than that for the earth (Barcilon 1976a, 1976b). I believe that the main result will remain valid even for the more complicated geophysical problem. Namely, if the source is known, then the amplitude and frequency response associated with a single horizontal wave number is sufficient to insure the uniqueness of the inverse problem.

I shall endeavor to present an overview of these results omitting most of the underlying calculations which either have appeared or will appear elsewhere.

2. Slab with Double-couple Excitation

Let us consider a perfectly elastic slab of infinite horizontal extent and of thickness a (see Fig.). We shall assume that the density ρ

Insert Figure.

Caption: Schematic diagram of the slab.

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and the Lamé parameters λ and μ are solely functions of the depth, i.e. of x. We shall also assume that the surface $x=0$ is stress-free, i.e.

$$p_{xx} = p_{x\xi_1} = p_{x\xi_2} = 0 \qquad \text{at } x = 0, \tag{2.1}$$

where $p_{..}$ stands for a component of the stress tensor. The slab is set in motion at time $t=0$ by means of a double-couple applied to the upper surface $x=a$. For the sake of presentation, we consider the simplest case possible and write

$$\left.\begin{aligned} p_{xx} &= 0 \\ p_{x\xi_1} &= -\,T\,\delta(\xi_1)\,\delta'(\xi_2)\,H(t) \\ p_{x\xi_2} &= -\,T\,\delta'(\xi_1)\,\delta(\xi_2)\,H(t) \end{aligned}\right\} \quad \text{at } x=a. \tag{2.2}$$

In the above formula, δ stands for the Dirac delta function and H for the Heaviside step-function; a prime denotes differentiation with respect to an appropriate variable.

It should already be clear that the model we are considering is closely related to the ultimate geophysical problem. The slab can be looked upon as the mantle of a flat earth without liquid core and gravitational force. To pursue the analogy with the geophysical situation, we

shall refer to the vertical and horizontal displacements $\underset{\sim}{u}(\underset{\sim}{a},\underset{\sim}{\xi},\underset{\sim}{t})$ and $\underset{\sim}{v}_\alpha(\underset{\sim}{a},\underset{\sim}{\xi},\underset{\sim}{t})$, $(\alpha=1,2)$, at the upper surface as the seismograms. The inverse problem can be stated thus: given these seismograms, can we determine $\underset{\sim}{\lambda}(\underset{\sim}{x})$, $\underset{\sim}{\mu}(\underset{\sim}{x})$ and $\underset{\sim}{\rho}(\underset{\sim}{x})$ uniquely?

3. Compressive and Torsional Modes

Just as for the earth, the response to an arbitrary excitation can be synthesized by means of a superposition of two kinds of normal modes of oscillations which we shall refer to as the compressive and torsional modes. In fact, in the spirit of Alterman et al (1959), let us write the displacements thus:

script "ach"

$$\underline{u}(\underline{x},\underline{\xi},\underline{t}) = \int_{-\infty}^{\infty} e^{\underline{-i\omega t}}\, d\underline{\omega} \iint_{\infty}^{\infty} \underline{y}_1(\underline{x},\underline{\omega};\underline{k})\, \mathcal{H}(\underline{\xi},\underline{k})\, d\underline{k},$$

$$\underline{v}_1(\underline{x},\underline{\xi},\underline{t}) = \int_{-\infty}^{\infty} e^{\underline{-i\omega t}}\, d\underline{\omega} \iint_{\infty}^{\infty} \{\underline{y}_3(\underline{x},\underline{\omega};\underline{k}) \frac{\partial \mathcal{H}}{\partial \underline{\xi}_1} + \underline{z}_1(\underline{x},\underline{\omega};\underline{k}) \frac{\partial \mathcal{H}}{\partial \underline{\xi}_2}\}\, d\underline{k}, \tag{3.1}$$

$$\underline{v}_2(\underline{x},\underline{\xi},\underline{t}) = \int_{-\infty}^{\infty} e^{\underline{-i\omega t}}\, d\underline{\omega} \iint_{-\infty}^{\infty} \{\underline{y}_3(\underline{x},\underline{\omega};\underline{k}) \frac{\partial \mathcal{H}}{\partial \underline{\xi}_2} - \underline{z}_1(\underline{x},\underline{\omega};\underline{k}) \frac{\partial \mathcal{H}}{\partial \underline{\xi}_1}\}\, d\underline{k}.$$

In the above representations

$$\mathcal{H}(\underline{\xi},\underline{k}) = \exp(-i\ \underline{\xi}\cdot\underline{k}) \tag{3.2}$$

is the function appropriate for the synthesis of the horizontal plan-form. Thus $\underline{k}=(\underline{k}_1,\underline{k}_2)$ is a horizontal wave number and it is convenient to define

$$\underline{\ell}^2 = \underline{k}_1^2 + \underline{k}_2^2\ . \tag{3.3}$$

$\underline{\omega}$ stands the frequency whereas the variables $\underline{y}_1$, $\underline{y}_3$ and $\underline{z}_1$ respectively are associated with the vertical structure of the displacements occuring in the compressive and torsional modes.

In addition to the displacement fields, we introduce the stress fields as follows:

$$\left.\begin{aligned} \underline{y}_2 &= (\underline{\lambda}+2\underline{\mu})\underline{y}_1' - \underline{\lambda}\,\ell^2\,\underline{y}_3, \\ \underline{y}_4 &= \underline{\mu}(\underline{y}_1+\underline{y}_3'), \end{aligned}\right\} \tag{3.4}$$

and

$$\underline{z}_2 = \underline{\mu}\,\underline{z}_1'. \tag{3.5}$$

Henceforth, primes will denote differentiation with respect to $\underline{x}$.

With these classical variables, the direct problem which consists in finding the slab's response to a double couple excitation can be formulated in terms of two boundary values for the $\underline{y}$- and $\underline{z}$- fields, namely

$$\begin{bmatrix} \underline{y}_1' \\ \underline{y}_2' \\ \underline{y}_3' \\ \underline{y}_4' \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1}{\underline{\lambda}+2\underline{\mu}} & \dfrac{\ell^2\underline{\lambda}}{\underline{\lambda}+2\underline{\mu}} & 0 \\ -\underline{\omega}^2\underline{\rho} & 0 & 0 & \ell^2 \\ -1 & 0 & 0 & \dfrac{1}{\underline{\mu}} \\ 0 & \dfrac{-\underline{\lambda}}{\underline{\lambda}+2\underline{\mu}} & -\underline{\omega}^2\underline{\rho}+4\underline{\mu}\dfrac{\underline{\lambda}+\underline{\mu}}{\underline{\lambda}+2\underline{\mu}}\ell^2 & 0 \end{bmatrix} \begin{bmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \underline{y}_3 \\ \underline{y}_4 \end{bmatrix} \tag{3.6a}$$

with

$$\left.\begin{aligned} &\underline{y}_2 = \underline{y}_4 = 0 && \text{at } \underline{x}=0, \\ &\underline{y}_2 = \underline{y}_4 + \frac{\underline{i}T}{8\underline{\pi}^3\underline{\omega}}\,\underline{\sin 2\theta} = 0 && \text{at } \underline{x}=\underline{a}; \end{aligned}\right\} \tag{3.6b}$$

and

$$\begin{bmatrix} \underset{\sim}{z}_1' \\ \underset{\sim}{z}_2' \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\underset{\sim}{\mu}} \\ -\underset{\sim}{\omega}^2 \underset{\sim}{\rho} + \underset{\sim}{\mu}\, \ell^2 & 0 \end{bmatrix} \begin{bmatrix} \underset{\sim}{z}_1 \\ \underset{\sim}{z}_2 \end{bmatrix} \tag{3.7a}$$

with

$$\left.\begin{array}{ll} \underset{\sim}{z}_2 = 0 & \text{at } \underset{\sim}{x}=0 \\ \underset{\sim}{z}_2 - \dfrac{iT}{8\underset{\sim}{\pi}^3 \underset{\sim}{\omega}} \underset{\sim}{\cos} 2\underset{\sim}{\theta} = 0 & \text{at } \underset{\sim}{x}=\underset{\sim}{a}. \end{array}\right\} \tag{3.7b}$$

In the above formulas, $(\ell, \underset{\sim}{\theta})$ are polar coordinates in the horizontal wave number space.

We shall be interested in the above equations in connection with the inverse problem for which we shall take the data to be $\underset{\sim}{y}_1(\underset{\sim}{a},\underset{\sim}{\omega};\underset{\sim}{k})$, $\underset{\sim}{y}_3(\underset{\sim}{a},\underset{\sim}{\omega};\underset{\sim}{k})$ and $\underset{\sim}{z}_1(\underset{\sim}{a},\underset{\sim}{\omega};\underset{\sim}{k})$, i.e. the temporal and spatial Fourier transforms of the seismograms. This statement will be made even more precise in sequel.

4. Properties of the Seismograms

Let $\eta_j^{(1)}$ and $\eta_j^{(2)}$ (i=1,2,3,4) be two fundamental solutions of (3.6a) such that

$$\left.\begin{aligned} \eta_j^{(1)} &= \delta_{1j} \\ \eta_j^{(2)} &= \delta_{3j} \end{aligned}\right\} \quad (j=1,2,3,4) \text{ for } x=0, \tag{4.1}$$

where δ_{ij} is the Kronecker delta. Clearly, $\eta_j^{(1)}$ and $\eta_j^{(2)}$ are functions of x, ω and ℓ. The solution of the boundary value problem (3.6) can be written in terms of these two solutions as follows:

$$y_j(x,\omega;\ell,\theta) = \frac{-iT}{8\pi^3\omega}\sin 2\theta\,\frac{\left[-\eta_2^{(2)}(a,\omega,\ell)\eta_j^{(1)}(x,\omega,\ell)+\eta_2^{(1)}(a,\omega,\ell)\eta_j^{(2)}(x,\omega,\ell)\right]}{W_{24}(a,\omega;\ell)} \tag{4.2}$$

$$(j=1,2,3,4)$$

where

$$W_{rs}(x,\omega;\ell) = \begin{vmatrix} \eta_r^{(1)}(x,\omega,\ell) & \eta_r^{(2)}(x,\omega,\ell) \\ \eta_s^{(1)}(x,\omega,\ell) & \eta_s^{(2)}(x,\omega,\ell) \end{vmatrix} . \tag{4.3}$$

As a result, the spatial and temporal Fourier transforms of the seismograms have the following forms:

$$y_1(a,\omega;\ell,\theta) = \frac{iT}{8\pi^3\omega}\sin 2\theta\,\frac{W_{12}(a,\omega;\ell)}{W_{24}(a,\omega;\ell)} , \tag{4.4a}$$

$$y_3(a,\omega;\ell,\theta) = -\frac{iT}{8\pi^3\omega}\sin 2\theta\,\frac{W_{23}(a,\omega;\ell)}{W_{24}(a,\omega;\ell)} . \tag{4.4b}$$

Looked upon as functions of ω, $W_{12}(a,\omega;\ell)$, $W_{23}(a,\omega;\ell)$ and $W_{24}(a,\omega;\ell)$ are completely determined by their zeros. Furthermore, these zeros are

natural frequencies of compressive modes of the slab in three vibrating configurations. These three configurations correspond respectively to the following boundary conditions at $\underset{\sim}{x}=\underset{\sim}{a}$;

$$\text{(i)} \quad \underset{\sim}{y}_1 = \underset{\sim}{y}_2 = 0 \tag{4.5a}$$

$$\text{(ii)} \quad \underset{\sim}{y}_2 = \underset{\sim}{y}_3 = 0 \tag{4.5b}$$

and

$$\text{(iii)} \quad \underset{\sim}{y}_2 = \underset{\sim}{y}_4 = 0 \; ; \tag{4.5c}$$

in all three cases, the boundary conditions at $\underset{\sim}{x}=0$ are $\underset{\sim}{y}_2 = \underset{\sim}{y}_4 = 0$. Thus, keeping ℓ fixed it is possible (at least in principle) to extract two "compressive" spectra over and above the compressive spectrum corresponding to stress free conditions at $\underset{\sim}{x}=0$, $\underset{\sim}{a}$.

Similarly, let us denote by $\underset{\sim}{\zeta}_\alpha(\underset{\sim}{x},\underset{\sim}{\omega};\ell)$ a solution of (3.7a) such that

$$\underset{\sim}{\zeta}_\alpha(0,\underset{\sim}{\omega};\ell) = \delta_{1\alpha} \qquad (\alpha=1,2). \tag{4.6}$$

Then, the solution of the boundary value problem (3.7) can be written thus

$$\underset{\sim}{z}_\alpha(\underset{\sim}{x},\underset{\sim}{\omega};\ell,\underset{\sim}{\theta}) = + \frac{\underset{\sim}{i}T}{8\underset{\sim}{\pi}^3\underset{\sim}{\omega}} \, \underset{\sim}{\cos 2\theta} \, \frac{\underset{\sim}{\zeta}_\alpha(\underset{\sim}{x},\underset{\sim}{\omega};\ell)}{\underset{\sim}{\zeta}_2(\underset{\sim}{a},\underset{\sim}{\omega};\ell)} \; . \tag{4.7}$$

In particular, that part of the spatial and temporal Fourier transforms of the seismogram associated with the torsional modes has the form

$$\underset{\sim}{z}_1(\underset{\sim}{a},\underset{\sim}{\omega};\ell,\underset{\sim}{\theta}) = \frac{\underset{\sim}{i}T}{8\underset{\sim}{\pi}^3\underset{\sim}{\omega}} \, \underset{\sim}{\cos 2\theta} \, \frac{\underset{\sim}{\zeta}_1(\underset{\sim}{a},\underset{\sim}{\omega};\ell)}{\underset{\sim}{\zeta}_2(\underset{\sim}{a},\underset{\sim}{\omega};\ell)} \; . \tag{4.8}$$

Once again, the zeros of $\underset{\sim}{\zeta}_1(\underset{\sim}{a},\underset{\sim}{\omega};\ell)$ and $\underset{\sim}{\zeta}_2(\underset{\sim}{a},\underset{\sim}{\omega};\ell)$ are the natural frequencies of the torsional modes of the slab in two vibrating configurations corresponding to the following boundary conditions at $\underset{\sim}{x}=\underset{\sim}{a}$

$$\text{(i)} \qquad \underset{\sim}{z}_1 = 0 \, , \tag{4.9a}$$

$$\text{(ii)} \qquad \underset{\sim}{z}_2 = 0 \, . \tag{4.9b}$$

The zeros of $\underset{\sim}{W}_{24}(\underset{\sim}{a},\underset{\sim}{\omega};\ell)$ and $\underset{\sim}{\zeta}_2(\underset{\sim}{a},\underset{\sim}{\omega};\ell)$ are associated with the frequency response of the slab whereas those of $\underset{\sim}{W}_{12}(\underset{\sim}{a},\underset{\sim}{\omega};\ell)$, $\underset{\sim}{W}_{23}(\underset{\sim}{a},\underset{\sim}{\omega};\ell)$ and $\underset{\sim}{\zeta}_1(\underset{\sim}{a},\underset{\sim}{\omega};\ell)$ are associated with the amplitude response.

5. Solution of the Inverse Problem

We can proceed along several routes. For instance, we could work with the so-called influence functions for a group of five eigenvalue problems (Barcilon 1976a). A second approach, particularly simple for discrete versions of these inverse problems, relies on continued fractions. This method has been pioneered by Krein (1952) and generalized by Barcilon (1976b). We shall refer the reader to these papers for a more discursive treatment of specific problems. Here, we shall confine ourselves to the derivation of the formulas essential for carrying out this procedure.

The first step consists in deriving differential equations for the six distinct Wronskians $\underline{W}_{ij}(\underline{x},\underline{\omega};\ell)$. These equations are

$$\underline{W}'_{12} = \ell^2 \underline{W}_{14} - \frac{\underline{\lambda}\ell^2}{\underline{\lambda}+2\underline{\mu}} \underline{W}_{23}, \tag{5.1a}$$

$$\underline{W}'_{13} = \frac{1}{\underline{\mu}} \underline{W}_{14} + \frac{1}{\underline{\lambda}+2\underline{\mu}} \underline{W}_{23}, \tag{5.1b}$$

$$\underline{W}'_{14} = -\frac{\underline{\lambda}}{\underline{\lambda}+2\underline{\mu}} \underline{W}_{12} + [-\underline{\omega}^2\underline{\rho} + 4\underline{\mu}\frac{\underline{\lambda}+\underline{\mu}}{\underline{\lambda}+2\underline{\mu}} \ell^2] \underline{W}_{13} + \frac{1}{\underline{\lambda}+2\underline{\mu}} \underline{W}_{24} + \frac{\underline{\lambda}\ell^2}{\underline{\lambda}+2\underline{\mu}} \underline{W}_{34}, \tag{5.1c}$$

$$\underline{W}'_{23} = \underline{W}_{12} - \underline{\omega}^2\underline{\rho}\, \underline{W}_{13} + \frac{1}{\underline{\mu}} \underline{W}_{24} - \ell^2 \underline{W}_{34}, \tag{5.1d}$$

$$\underline{W}'_{24} = -\underline{\omega}^2\underline{\rho}\, \underline{W}_{14} + [-\underline{\omega}^2\underline{\rho} + 4\underline{\mu}\frac{\underline{\lambda}+\underline{\mu}}{\underline{\lambda}+2\underline{\mu}} \ell^2] \underline{W}_{23}, \tag{5.1e}$$

$$\underline{W}'_{34} = -\underline{W}_{14} + \frac{\underline{\lambda}}{\underline{\lambda}+2\underline{\mu}} \underline{W}_{23} . \tag{5.1f}$$

Actually, from (5.1a) and (5.1f) together with (4.1) we deduce that $\underline{W}_{34}$ and $\underline{W}_{12}$ are linearly related, viz.

$$\underline{W}_{12}(\underline{x},\underline{\omega};\ell) + \ell^2\underline{W}_{34}(\underline{x},\underline{\omega};\ell) = 0. \tag{5.2}$$

The Wronskians satisfy also the following identify

$$\underline{W}_{12}\,\underline{W}_{34} + \underline{W}_{13}\,\underline{W}_{42} + \underline{W}_{14}\,\underline{W}_{23} = 0\ ,$$

or in view of (5.2)

$$\underline{W}_{12}^2(\underline{x}) + \ell^2\underline{W}_{13}(\underline{x})\underline{W}_{24}(\underline{x}) - \ell^2\underline{W}_{14}(\underline{x})\underline{W}_{23}(\underline{x}) = 0. \tag{5.3}$$

By eliminating $\underline{W}_{34}$ also from (5.1) and incorporating the equations (3.7a) for $\underline{\zeta}$, we obtain the following grand system of first order equations:

$$\begin{bmatrix} \underline{W}'_{12} \\ \underline{W}'_{13} \\ \underline{W}'_{14} \\ \underline{W}'_{23} \\ \underline{W}'_{24} \\ \underline{\zeta}'_1 \\ \underline{\zeta}'_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \ell^2 & \frac{-\underline{\lambda}\ell^2}{\underline{\lambda}+2\underline{\mu}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\underline{\mu}} & \frac{1}{\underline{\lambda}+2\underline{\mu}} & 0 & 0 & 0 \\ -\frac{2\underline{\lambda}}{\underline{\lambda}+2\underline{\mu}} & -\underline{\omega}^2\underline{\rho}+4\frac{\underline{\lambda}+\underline{\mu}}{\underline{\lambda}+2\underline{\mu}}\ell^2 & 0 & 0 & \frac{1}{\underline{\lambda}+2\underline{\mu}} & 0 & 0 \\ 2 & -\underline{\omega}^2\underline{\rho} & 0 & 0 & \frac{1}{\underline{\mu}} & 0 & 0 \\ 0 & 0 & -\underline{\omega}^2\underline{\rho} & -\underline{\omega}^2\underline{\rho}+4\underline{\mu}\frac{\underline{\lambda}+\underline{\mu}}{\underline{\lambda}+2\underline{\mu}}\ell^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\underline{\mu}} \\ 0 & 0 & 0 & 0 & 0 & -\underline{\omega}^2\underline{\rho}+\underline{\mu}\ell^2 & 0 \end{bmatrix} \begin{bmatrix} \underline{W}_{12} \\ \underline{W}_{13} \\ \underline{W}_{14} \\ \underline{W}_{23} \\ \underline{W}_{24} \\ \underline{\zeta}_1 \\ \underline{\zeta}_2 \end{bmatrix} \tag{5.4}$$

If the slab is discretized in N layers, then $\underline{W}_{\underline{j}\underline{k}}(\underline{a},\underline{\omega})$ and $\underline{\zeta}_\alpha(\underline{a},\underline{\omega})$ are known polynomials of degree N in $\underline{\omega}^2$. Indeed, $\underline{W}_{12}(\underline{a},\underline{\omega})$, $\underline{W}_{23}(\underline{a},\underline{\omega})$, $\underline{W}_{24}(\underline{a},\underline{\omega})$, $\underline{\zeta}_1(\underline{a},\underline{\omega})$ and $\underline{\zeta}_2(\underline{a},\underline{\omega})$ are extracted from the seismograms, whereas $\underline{W}_{13}(\underline{a},\underline{\omega})$ and $\underline{W}_{14}(\underline{a},\underline{\omega})$ are deduced from (5.3). Furthermore the $\underline{W}$'s and $\underline{\zeta}$'s are also known at $\underline{x}=0$, viz

$$\left.\begin{array}{c}\underline{W}_{12}(0,\underline{\omega}) = \underline{W}_{14}(0,\underline{\omega}) = \underline{W}_{23}(0,\underline{\omega}) = \underline{W}_{24}(0,\underline{\omega}) = \underline{\zeta}_2(0,\underline{\omega}) = 0, \\ \underline{W}_{13}(0,\underline{\omega}) = \underline{\zeta}_1(0,\underline{\omega}) = 1.\end{array}\right\} \qquad (5.5)$$

With these informations, we can find $\underline{\lambda}$, $\underline{\mu}$ and $\underline{\rho}$ by repeating the steps indicated in Barcilon (1967b).

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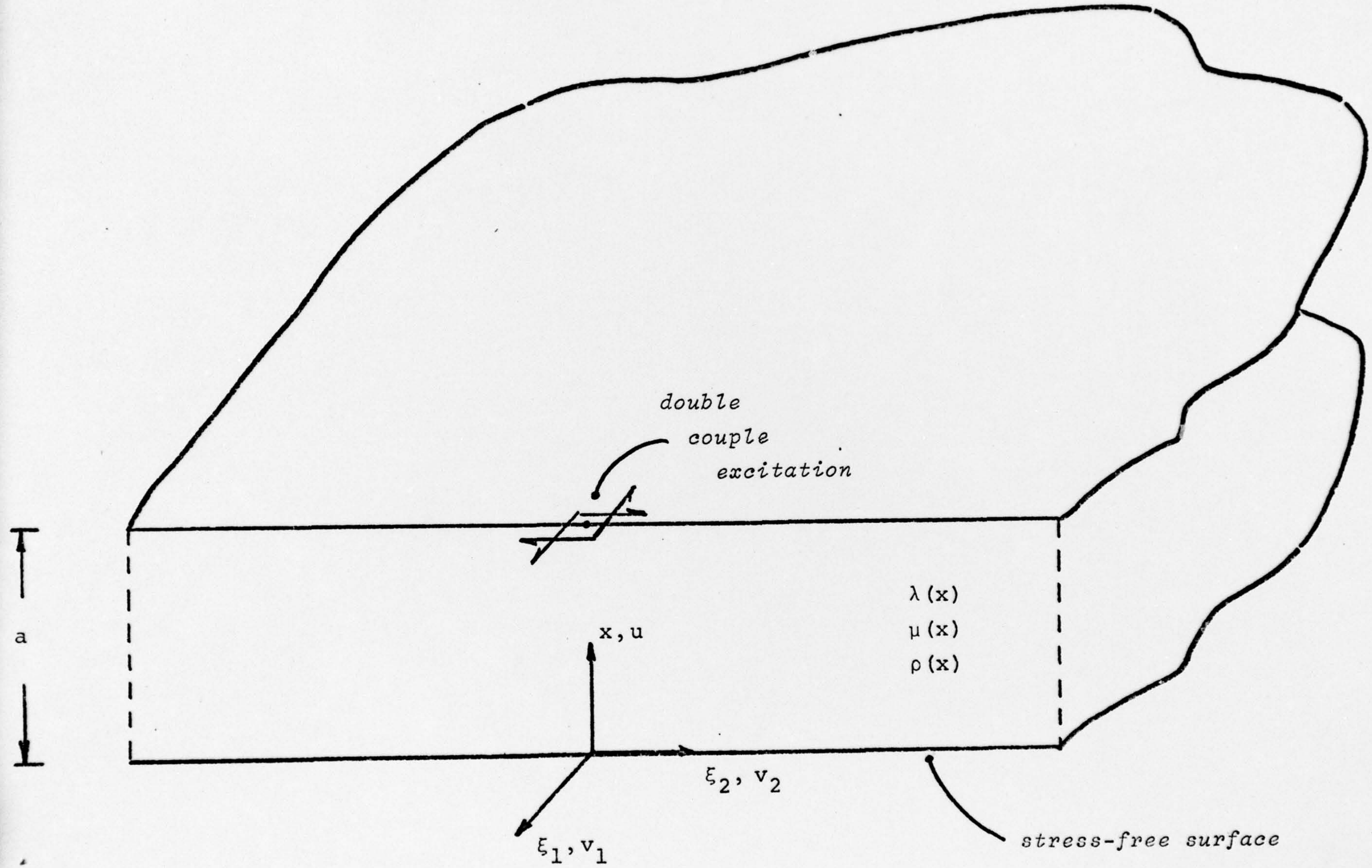
References

Alterman, Z., Jarosch, H., Pekeris, C. L.: Oscillations of the earth. Proc. Roy. Soc. London, A, 252, 80-95, 1959.

Barcilon, V.: Inverse problem for a vibrating beam. ZAMP, 27, 347-358, 1976a.

Barcilon, V.: On the solution of the inverse problem with amplitude and natural frequency data. Phys. Earth Planetary Int., (in press), 1976b.

Krein, M. G.: Ob obratnykh zadachakh dlya neodnorodnoi struny. Dokl. Akad. Nauk SSSR, 82, 669-672, 1952.


double couple excitation
a
x, u
λ(x)
μ(x)
ρ(x)
ξ2, v2
ξ1, v1
stress-free surface